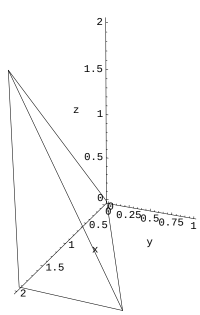
1. **Reference**: Problem 11.7.8, from the assignment due March 10, and Quiz 2.

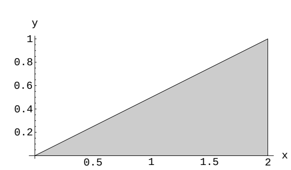
$$\begin{array}{rcl}
f_x & = & 6x^2 + 3y^2 + 18x \\
f_y & = & 6xy + 12y \\
 & = & 6y(x+2) \Longrightarrow y = 0 \text{ or } x = -2 \\
\text{If } y = 0: \quad f_x & = & 6x^2 + 18x = 0 \\
6x(x+3) & = & 0 \Longrightarrow x = 0 \text{ or } x = -3 \\
 & & \boxed{(0,0)\,,(-3,0)} \\
\text{If } x = -2: \quad f_x & = & 24 + 3y^2 - 36 = 0 \\
3y^2 & = & 12 \Longrightarrow y = \pm 2 \\
 & & \boxed{(-2,-2)\,,(-2,2)} \\
f_{xx} & = & 12x + 18 \\
f_{yy} & = & 6x + 12 \\
f_{xy} & = & 6y \\
D & = & (12x+18)(6x+12) - (6y)^2 \\
 & = & 36[(2x+3)(x+2) - y^2] \qquad \{\text{We can ignore the 36.} \qquad \} \\
D(0,0) & = & 6, f_{xx} = 18 \Longrightarrow \boxed{(0,0) \text{ is a min.}} \\
D(-3,0) & = & 3, f_{xx} = -18 \Longrightarrow \boxed{(-3,0) \text{ is a max.}} \\
D(-2,-2) & = & -4 \Longrightarrow \boxed{(-2,-2) \text{ is a saddle.}} \\
D(-2,2) & = & -4, \Longrightarrow \boxed{(-2,2) \text{ is a saddle.}}
\end{array}$$

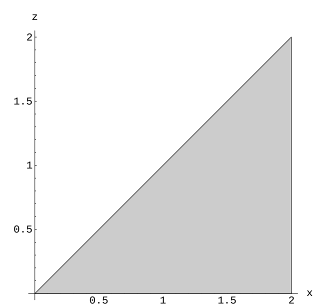
2. **Reference**: Problems 12.2.36 from the homework due Wednesday, March 17, and Quiz 2.
   Graph the region and switch the order of integration:

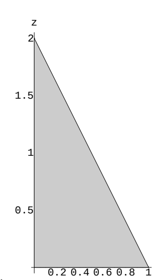
$$\begin{array}{rcl}
\displaystyle\int_{y=0}^{y=\frac{\pi}{3}} \int_{x=0}^{x=\tan y} \sec^3 y \, dx \, dy & = & \displaystyle\int_{y=0}^{y=\frac{\pi}{3}} \tan y \sec^3 y \, dy \qquad \{\text{Let } u = \sec y, du = \sec y \tan y \, dy.\} \\
 & = & \displaystyle\int u^2 \, du \\
 & = & \dfrac{u^3}{3} \\
 & = & \dfrac{1}{3} \sec^3 y \Big|_{y=0}^{y=\frac{\pi}{3}} \\
 & = & \dfrac{1}{3}\left(2^3 - 1^3\right) \\
 & = & \boxed{\dfrac{7}{3}}
\end{array}$$

3. **Reference**: Problem 12.5.28, from the homework due Wednesday, March 24.









By looking at the shadows on the three planes we get

$$\boxed{\int_{x=0}^{x=2}\int_{y=0}^{y=\frac{x}{2}}\int_{z=0}^{z=x-2y} f(x,y,z)\,dz\,dy\,dx} = \boxed{\int_{y=0}^{y=1}\int_{x=2y}^{x=2}\int_{z=0}^{z=x-2y} f(x,y,z)\,dz\,dx\,dy}$$
$$= \boxed{\int_{y=0}^{y=1}\int_{z=0}^{z=2-2y}\int_{x=z+2y}^{x=2} f(x,y,z)\,dx\,dz\,dy} = \boxed{\int_{z=0}^{z=2}\int_{y=0}^{y=1-\frac{z}{2}}\int_{x=z+2y}^{x=2} f(x,y,z)\,dx\,dy\,dz}$$
$$= \boxed{\int_{x=0}^{x=2}\int_{z=0}^{z=x}\int_{y=0}^{y=\frac{x-z}{2}} f(x,y,z)\,dy\,dz\,dx} = \boxed{\int_{z=0}^{z=2}\int_{x=z}^{x=2}\int_{y=0}^{y=\frac{x-z}{2}} f(x,y,z)\,dy\,dx\,dz}.$$

4. **Reference**: Problem 10.1.30, from the homework due Wednesday, January 27, Problem 12.7.27, similar to the homework due Wednesday, April 7, and Example 4 on page 710 (all mentioned in the review).

   Convert to spherical coordinates. The cone is $\varphi = \frac{\pi}{4}$ and the equation of the sphere converts as follows:

$$\begin{aligned} \rho^2 &= 2\rho\cos\varphi \\ \rho &= 2\cos\varphi \end{aligned}$$

   When you graph the region, you can see the following limits:

$$\boxed{\begin{array}{lll} 0 & \leq \varphi \leq & \frac{\pi}{4} \\ 0 & \leq \theta \leq & 2\pi \\ 0 & \leq \rho \leq & 2\cos\varphi \end{array}}$$

   The function converts to $f(\rho,\theta,\varphi) = \frac{1}{\rho^2}$.

$$\begin{aligned} \int_{\varphi=0}^{\varphi=\frac{\pi}{4}}\int_{\theta=0}^{\theta=2\pi}\int_{\rho=0}^{\rho=2\cos\varphi} \frac{1}{\rho^2}\rho^2\sin\varphi\,d\rho\,d\theta\,d\varphi &= \int_{\varphi=0}^{\varphi=\frac{\pi}{4}}\int_{\theta=0}^{\theta=2\pi}\int_{\rho=0}^{\rho=2\cos\varphi} \sin\varphi\,d\rho\,d\theta\,d\varphi \\ &= \int_{\varphi=0}^{\varphi=\frac{\pi}{4}}\int_{\theta=0}^{\theta=2\pi} 2\cos\varphi\sin\varphi\,d\theta\,d\varphi \\ &= 4\pi\int_{\varphi=0}^{\varphi=\frac{\pi}{4}} \cos\varphi\sin\varphi\,d\varphi \qquad \{\text{Let } u = \sin\varphi, du = \cos\varphi\,d\varphi \quad\} \\ &= 2\pi\sin^2\varphi\Big|_{\varphi=0}^{\varphi=\frac{\pi}{4}} \\ &= 2\pi\left(\frac{\sqrt{2}}{2}\right)^2 \\ &= \boxed{\pi} \end{aligned}$$

5. **Reference**: Problem 12.8.15, from the homework due Wednesday, April 7.

$$J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} \frac{1}{v} & -\frac{u}{v^2} \\ 0 & 1 \end{vmatrix} = \frac{1}{v}$$

   $1 \leq xy \leq 4$ translates to $1 \leq u \leq 4$ and $x \leq y \leq 4x$ translates to $\sqrt{u} \leq v \leq 2\sqrt{u}$.

$$\begin{aligned} \int_{u=1}^{u=4}\int_{v=\sqrt{u}}^{v=2\sqrt{u}} u\cdot\frac{1}{v}\,dv\,du &= \int_{u=1}^{u=4} u\ln v\Big|_{v=\sqrt{u}}^{v=2\sqrt{u}}\,du \\ &= \int_{u=1}^{u=4} u(\ln(2\sqrt{u}) - \ln\sqrt{u})\,du \\ &= \int_{u=1}^{u=4} u\ln 2\,du \\ &= \frac{u^2}{2}\ln 2\Big|_{u=1}^{u=4} \\ &= \boxed{\frac{15\ln 2}{2}} \end{aligned}$$